Introduction to Trigonometry

Selected NCERT Questions

1. In Fig. 9.6, find tan P – cot R.

Sol. Using Pythagoras Theorem, we have

$$PR^2 = PQ^2 + QR^2$$

$$\Rightarrow \quad (13)^2 = (12)^2 + QR^2$$

$$\Rightarrow \quad 169 = 144 + QR^2$$

$$\Rightarrow \quad QR^2 = 169 - 144 = 25 \quad \Rightarrow \quad QR = 5 \text{ cm}$$

Now, $\tan P = \dfrac{QR}{PQ} = \dfrac{5}{12}$ and $\cot R = \dfrac{QR}{PQ} = \dfrac{5}{12}$

$$\therefore \quad \tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$

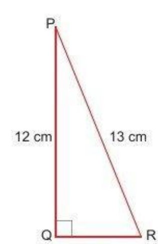

Fig. 9.6

2. In triangle ABC right-angled at B, if $\tan A = \frac{1}{\sqrt{3}}$ find the value of:

(i) $\sin A \cos C + \cos A \sin C$ (ii) $\cos A \cos C - \sin A \sin C$.

Sol. We have a right-angled ΔABC in which $\angle B = 90°$ and $\tan A = \frac{1}{\sqrt{3}}$

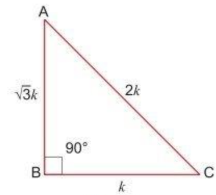

Fig. 9.7

Now, $\tan A = \frac{1}{\sqrt{3}} = \frac{BC}{AB}$

Let $BC = k$ and $AB = \sqrt{3}k$

$\therefore$ By Pythagoras Theorem, we have

$\Rightarrow$ $AC^2 = AB^2 + BC^2$

$\Rightarrow$ $AC^2 = (\sqrt{3}k)^2 + (k)^2 = 3k^2 + k^2$

$\Rightarrow$ $AC^2 = 4k^2$ $\Rightarrow$ $AC = 2k$

Now, $\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{k}{2k} = \frac{1}{2}$; $\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$; $\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{k}{2k} = \frac{1}{2}$

(i) $\sin A \,.\, \cos C + \cos A \,.\, \sin C = \frac{1}{2}\times\frac{1}{2}+\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} = \frac{1}{4}+\frac{3}{4} = \frac{4}{4} = 1$

(ii) $\cos A \,.\, \cos C - \sin A \,.\, \sin C = \frac{\sqrt{3}}{2}\times\frac{1}{2}-\frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4} = 0$.

3. In ΔPQR, right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

Sol. We have a right-angled ΔPQR in which $\angle Q = 90°$.

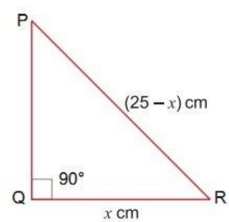

Fig. 9.8

Let $QR = x$ cm

Therefore, $PR = (25 - x)$ cm

By Pythagoras Theorem, we have

$PR^2 = PQ^2 + QR^2$

$\Rightarrow$ $(25 - x)^2 = 5^2 + x^2$

$\Rightarrow$ $(25 - x)^2 - x^2 = 25$

$\Rightarrow$ $(25 - x - x)(25 - x + x) = 25$

$\Rightarrow$ $(25 - 2x)\, 25 = 25$

$\Rightarrow$ $25 - 2x = 1$

$\Rightarrow$ $25 - 1 = 2x$ $\Rightarrow$ $24 = 2x$

$\therefore$ $x = 12$ cm

Hence, $QR = 12$ cm

$PR = (25 - x)$ cm $= 25 - 12 = 13$ cm

$PQ = 5$ cm

$\therefore$ $\sin P = \frac{QR}{PR} = \frac{12}{13}$; $\cos P = \frac{PQ}{PR} = \frac{5}{13}$; $\tan P = \frac{QR}{PQ} = \frac{12}{5}$

4. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$; $0° < A + B \le 90°$; $A > B$, find A and B.

Sol. We have, $\tan(A+B) = \sqrt{3} \Rightarrow \tan(A+B) = \tan 60°$

$\therefore \quad A + B = 60° \quad \ldots(i)$

Again, $\tan(A-B) = \frac{1}{\sqrt{3}} \Rightarrow \tan(A-B) = \tan 30°$

$\therefore \quad A - B = 30° \quad \ldots(ii)$

Adding (*i*) and (*ii*), we have

$$2A = 90° \Rightarrow A = 45°$$

Putting the value of A in (*i*), we have

$$45° + B = 60°$$

$\therefore \quad B = 60° - 45° = 15°$

Hence, $A = 45°$ and $B = 15°$.

5. Prove that: $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

[*CBSE* 2019 (30/1/2)]

Sol. LHS $= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \cdot \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \cdot \sec A$

$= (\sin^2 A + \operatorname{cosec}^2 A + 2) + (\cos^2 A + \sec^2 A + 2)$ ($\because \sin A \cdot \operatorname{cosec} A = 1$ & $\cos A \cdot \sec A = 1$)

$= (\sin^2 A + \cos^2 A) + (\operatorname{cosec}^2 A + \sec^2 A) + 4$

$= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 4$ ($\because 1 + \cot^2 A = \operatorname{cosec}^2 A$ & $1 + \tan^2 A = \sec^2 A$)

$= 7 + \tan^2 A + \cot^2 A$

$=$ RHS

6. Prove that: $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

Sol. LHS $= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A} = \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{(\cos^2 A + \sin^2 A) + 1 + 2\sin A}{(1+\sin A)\cos A} = \frac{1 + 1 + 2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{2(1+\sin A)}{(1+\sin A)\cos A} = \frac{2}{\cos A} = 2\sec A = \text{RHS}.$$

7. Prove that: $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$ [*CBSE* 2020 (30/5/1)]

Sol. LHS $= \sqrt{\frac{1+\sin A}{1-\sin A}} = \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$ (By rationalisation)

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}} = \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A = \text{RHS}$$

Hence Proved.

8. Prove the following identity, where the angle involved is acute angle for which the expressions are defined.

$$\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A, \text{ using the identity } \operatorname{cosec}^2 A = 1 + \cot^2 A.$$

[*Competency Based Question*]

Sol. LHS $= \dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \dfrac{\dfrac{\cos A - \sin A + 1}{\sin A}}{\dfrac{\cos A + \sin A - 1}{\sin A}} = \dfrac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$

$= \dfrac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1}$ $\quad [\because \operatorname{cosec}^2 A - \cot^2 A = 1]$

$= \dfrac{(\cot A + \operatorname{cosec} A) - [(\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)]}{\cot A - \operatorname{cosec} A + 1}$

$= \dfrac{(\operatorname{cosec} A + \cot A)(1 - \operatorname{cosec} A + \cot A)}{(\cot A - \operatorname{cosec} A + 1)}$

$= \operatorname{cosec} A + \cot A$

$=$ RHS.

9. Prove that : $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$ **(θ is replaced by A)** [*CBSE* 2018 (30/1)]

Sol.

27) To prove: $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$.

Simplifying LHS,

$\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$= \dfrac{\sin A\,(1 - 2\sin^2 A)}{\cos A\,(2\cos^2 A - 1)}$

$= \dfrac{\sin A}{\cos A}\left[\dfrac{1 - (2\sin^2 A)}{2\cos^2 A - 1}\right]$

$= \dfrac{\sin A}{\cos A}\left[\dfrac{\sin^2 A + \cos^2 A - 2\sin^2 A}{2\cos^2 A - (\sin^2 A + \cos^2 A)}\right]$ $\quad [\because \sin^2 A + \cos^2 A = 1]$

$= \dfrac{\sin A}{\cos A}\left[\dfrac{\cos^2 A - \sin^2 A}{\cos^2 A - \sin^2 A}\right]$

$= \dfrac{\sin A}{\cos A} \times 1$

$= \tan A.$ $\quad \left[\because \dfrac{\sin A}{\cos A} = \tan A\right]$

LHS = RHS

hence proved.

[*Topper's Answer* 2018]

10. Prove that: $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta=1+\tan\theta+\cot\theta$

[Competency Based Question]

Sol. LHS $=\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=\dfrac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\dfrac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$=\dfrac{\sin\theta\times\sin\theta}{\cos\theta(\sin\theta-\cos\theta)}+\dfrac{\cos\theta}{\sin\theta}\times\dfrac{\cos\theta}{(\cos\theta-\sin\theta)}$

$=\dfrac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\dfrac{\cos^2\theta}{\sin\theta\{-(\sin\theta-\cos\theta)\}}$

$=\dfrac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\dfrac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}=\dfrac{\sin^3\theta-\cos^3\theta}{\cos\theta(\sin\theta-\cos\theta)\sin\theta}$

$=\dfrac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$

$=\dfrac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$ ...(*i*)

$=\dfrac{1}{\sin\theta\cos\theta}+\dfrac{\sin\theta\cos\theta}{\sin\theta\cos\theta}=\dfrac{1}{\sin\theta}\cdot\dfrac{1}{\cos\theta}+1$

$=\sec\theta\operatorname{cosec}\theta+1=$ RHS

From (*i*), $\dfrac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}=\dfrac{\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$=\dfrac{\sin^2\theta}{\sin\theta\cos\theta}+\dfrac{\cos^2\theta}{\sin\theta\cos\theta}+\dfrac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$=\tan\theta+\cot\theta+1=$ RHS

11. Prove that: $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

Sol. LHS $=\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right)=\dfrac{\sec^2 A}{\operatorname{cosec}^2 A}$

$=\dfrac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}=\dfrac{\sin^2 A}{\cos^2 A}=\tan^2 A$

RHS $=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\left(\dfrac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$

$=\left(\dfrac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2=\left(\dfrac{1-\tan A}{\tan A-1}\times\tan A\right)^2$

$=(-\tan A)^2=\tan^2 A$

LHS = RHS.

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. **Consider the triangle shown below.**

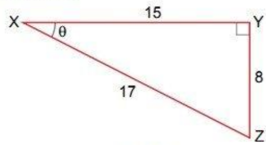

Fig. 9.9

What are the values of tan θ, cosec θ and sec θ?

(*a*) $\tan\theta = \frac{8}{15}, \operatorname{cosec}\theta = \frac{17}{8}, \sec\theta = \frac{17}{15}$ (*b*) $\tan\theta = \frac{8}{15}, \operatorname{cosec}\theta = \frac{17}{15}, \sec\theta = \frac{17}{8}$

(*c*) $\tan\theta = \frac{17}{15}, \operatorname{cosec}\theta = \frac{8}{15}, \sec\theta = \frac{17}{8}$ (*d*) $\tan\theta = \frac{8}{15}, \operatorname{cosec}\theta = \frac{17}{15}, \sec\theta = \frac{8}{17}$

2. **If $\sin A = \frac{1}{2}$, then the value of $\cot A$ is** [*NCERT Exemplar*]

(*a*) $\sqrt{3}$ (*b*) $\frac{1}{\sqrt{3}}$ (*c*) $\frac{\sqrt{3}}{2}$ (*d*) 1

3. **The two legs *AB* and *BC* of right triangle *ABC* are in a ratio 1 : 3. What will be the value of sin *C*?**

(*a*) $\sqrt{10}$ (*b*) $\frac{1}{\sqrt{10}}$ (*c*) $\frac{3}{\sqrt{10}}$ (*d*) $\frac{1}{2}$

4. **If $\sin A + \sin^2 A = 1$, then the value of the expression $(\cos^2 A + \cos^4 A)$ is** [*NCERT Exemplar*]

(*a*) 1 (*b*) $\frac{1}{2}$ (*c*) 2 (*d*) 3

5. **Which of these is equivalent to $\frac{2\tan x\,(\sec^2 x - 1)}{\cos^3 x}$?** [*Competency Based Question*]

(*a*) $2\tan^3 x \operatorname{cosec} x$ (*b*) $2\cot^3 x \operatorname{cosec}^3 x$ (*c*) $2\tan^3 x \sec^3 x$ (*d*) $2\cot^3 x \sec^3 x$

6. **If $4\tan\theta = 3$, then $\left(\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$ is equal to** [*NCERT Exemplar*]

(*a*) $\frac{2}{3}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{3}{4}$

7. **What is the value of $\frac{3 - \sin^2 60°}{\tan 30° \tan 60°}$?**

(*a*) $2\frac{1}{4}$ (*b*) $3\frac{1}{4}$ (*c*) $2\frac{3}{4}$ (*d*) $3\frac{3}{4}$

8. **The value of $\frac{4 - \sin^2 45°}{\cot k \tan 60°}$ is 3.5. What is the value of *k*?**

(*a*) 30° (*b*) 45° (*c*) 60° (*d*) 90°

9. **The value of $\frac{2\tan 30°}{1 - \tan^2 30°}$ is equal to**

(*a*) cos 60° (*b*) sin 60° (*c*) tan 60° (*d*) sin 30°

10. **The value of θ for which $\cos(10° + \theta) = \sin 30°$, is** [*CBSE 2020 (30/4/1)*]

(*a*) 50° (*b*) 40° (*c*) 80° (*d*) 20°

11. Given that $\sin\theta = \frac{a}{b}$, then $\cos\theta$ is equal to [*NCERT Exemplar*]

(*a*) $\frac{b}{\sqrt{b^2 - a^2}}$ (*b*) $\frac{b}{a}$ (*c*) $\frac{\sqrt{b^2 - a^2}}{b}$ (*d*) $\frac{a}{\sqrt{b^2 - a^2}}$

12. If $x = r\sin\theta$ and $y = r\cos\theta$ then the value of $x^2 + y^2$ is

(*a*) r (*b*) r^2 (*c*) $\frac{1}{r}$ (*d*) 1

13. If $\tan x + \sin x = m$ and $\tan x - \sin x = n$ then $m^2 - n^2$ is equal to

(*a*) $4\sqrt{mn}$ (*b*) $\sqrt{mn}$ (*c*) $2\sqrt{mn}$ (*d*) none of them

14. The value of θ for which $\sin(44^\circ + \theta) = \cos 30^\circ$, is [*CBSE* 2020 (30/4/3)]

(*a*) 46° (*b*) 60° (*c*) 16° (*d*) 90°

Answers

1. (*a*)	2. (*a*)	3. (*b*)	4. (*a*)	5. (*c*)	6. (*c*)	7. (*a*)
8. (*c*)	9. (*c*)	10. (*a*)	11. (*c*)	12. (*b*)	13. (*a*)	14. (*c*)

Very Short Answer Questions

Each of the following questions are of 1 mark.

1. What is the value of $\left(\frac{1}{1+\cot^2\theta} + \frac{1}{1+\tan^2\theta}\right)$? [*CBSE* 2020 (30/3/1)]

Sol. We have, $\left(\frac{1}{1+\cot^2\theta} + \frac{1}{1+\tan^2\theta}\right)$

$$= \left(\frac{1}{\operatorname{cosec}^2\theta} + \frac{1}{\sec^2\theta}\right) = \sin^2\theta + \cos^2\theta = 1$$

2. If $\tan\alpha = \frac{5}{12}$, find the value of $\sec\alpha$? [*CBSE* 2019 (30/3/2)]

Sol.

2. $\tan\alpha = \frac{5}{12}$

Using identity; $\sec^2\alpha - \tan^2\alpha = 1$

$\sec^2\alpha = 1 + \tan^2\alpha$

$\Rightarrow \sec^2\alpha = 1 + \left(\frac{5}{12}\right)^2$

$= 1 + \frac{25}{144}$

$= \frac{144+25}{144}$

$\Rightarrow \sec^2\alpha = \frac{169}{144} \Rightarrow \sec\alpha = \sqrt{\frac{13^2}{12^2}}$

$\sec\alpha = \frac{13}{12}$

[*Topper's Answer* 2019]

3. **If $\sec^2\theta(1+\sin\theta)(1-\sin\theta) = k$, then find the value of k.**

Sol. We have, $\sec^2\theta(1+\sin\theta)(1-\sin\theta) = \sec^2\theta(1-\sin^2\theta)$ $(\because (a+b)(a-b) = a^2 - b^2)$

$= \sec^2\theta.\cos^2\theta = 1$ $(\because \cos^2\theta + \sin^2\theta = 1)$

$\therefore \quad k = 1$

4. **If $\sin\theta = \frac{1}{3}$, then find the value of $2\cot^2\theta + 2$.**

Sol. $2\cot^2\theta + 2 = 2(\cot^2\theta + 1) = 2\text{cosec}^2\theta = \frac{2}{\sin^2\theta} = \frac{2}{\left(\frac{1}{3}\right)^2} = 2\times 9 = 18$

5. **Evaluate: $\frac{2\tan 45°\times\cos 60°}{\sin 30°}$** [*CBSE* 2020 (30/5/1)]

Sol. We have,

$$\frac{2\tan 45°\times\cos 60°}{\sin 30°} = \frac{2\times 1\times\frac{1}{2}}{\frac{1}{2}} = 2$$

6. **Write the acute angle θ satisfying $\sqrt{3}\sin\theta = \cos\theta$.**

Sol. We have, $\sqrt{3}\sin\theta = \cos\theta$

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}} \quad\Rightarrow\quad \tan\theta = \frac{1}{\sqrt{3}} \quad\Rightarrow\quad \theta = 30°$

7. **If $\sin x + \cos y = 1$; $x = 30°$ and y is an acute angle, find the value of y.** [*CBSE* 2019 (30/5/1)]

Sol. We have, $\sin x + \cos y = 1$

$\Rightarrow \quad \sin 30° + \cos y = 1$

$\Rightarrow \quad \frac{1}{2} + \cos y = 1 \quad\Rightarrow\quad \cos y = 1 - \frac{1}{2} = \frac{1}{2} \quad\Rightarrow\quad y = 60°$

Short Answer Questions–I

Each of the following questions are of 2 marks.

1. **Prove that $1 + \frac{\cot^2\alpha}{1+\text{cosec}\,\alpha} = \text{cosec}\,\alpha$.**

[*NCERT Exemplar, CBSE* 2019(*C*) (30/1/1), *CBSE* 2020 (30/2/1)]

Sol. LHS $= 1 + \frac{\cot^2\alpha}{1+\text{cosec}\,\alpha}$

$= 1 + \frac{\text{cosec}^2\alpha - 1}{1+\text{cosec}\,\alpha}$

$= 1 + \frac{(\text{cosec}\,\alpha - 1)(\text{cosec}\,\alpha + 1)}{(1+\text{cosec}\,\alpha)}$

$= 1 + \text{cosec}\,\alpha - 1$

$= \text{cosec}\,\alpha$ = RHS   Proved

2. **Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$.** [*CBSE* 2020 (30/2/1)]

Sol. LHS $= \tan^4\theta + \tan^2\theta$

$= \tan^2\theta(\tan^2\theta + 1)$ ½

$= (\sec^2\theta - 1)(\sec^2\theta) = \sec^4\theta - \sec^2\theta$ = RHS   1+½

[*CBSE Marking Scheme* 2020 (30/2/1)]

3. **Given that $\sin\theta = \frac{a}{b}$, find the value of $\tan\theta$.**

Sol. $\sin\theta = \frac{a}{b}$

$\Rightarrow \quad \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{a^2}{b^2}} = \sqrt{\frac{b^2-a^2}{b^2}} = \frac{\sqrt{b^2-a^2}}{b}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{a/b}{\frac{\sqrt{b^2-a^2}}{b}} = \frac{a}{\sqrt{b^2-a^2}}$$

4. **Prove that $(\sin\alpha+\cos\alpha)(\tan\alpha+\cot\alpha) = \sec\alpha+\operatorname{cosec}\alpha$.** *[NCERT Exemplar]*

Sol. LHS = $(\sin\alpha + \cos\alpha)(\tan\alpha + \cot\alpha)$

$$= (\sin\alpha+\cos\alpha)\left(\frac{\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{\sin\alpha}\right)$$

$$= (\sin\alpha+\cos\alpha)\left(\frac{\sin^2\alpha+\cos^2\alpha}{\cos\alpha\sin\alpha}\right)$$

$$= (\sin\alpha+\cos\alpha)\times\frac{1}{\cos\alpha\,.\,\sin\alpha}$$

$$= \frac{\sin\alpha}{\cos\alpha\sin\alpha}+\frac{\cos\alpha}{\cos\alpha\sin\alpha}$$

$$= \frac{1}{\cos\alpha}+\frac{1}{\sin\alpha} = \sec\alpha+\operatorname{cosec}\alpha = \text{RHS}$$

5. **If $\tan\theta = \frac{3}{4}$, find the value of $\left(\frac{1-\cos^2\theta}{1+\cos^2\theta}\right)$.** *[CBSE 2020 (30/3/1)]*

Sol. Given, $\tan\theta = \frac{3}{4}$

Since $\sec\theta = \sqrt{1+\tan^2\theta}$

$\Rightarrow \quad \sec\theta = \sqrt{1+\left(\frac{3}{4}\right)^2} = \sqrt{1+\frac{9}{16}} = \sqrt{\frac{16+9}{16}}$

$\Rightarrow \quad \sec\theta = \frac{5}{4} \quad \Rightarrow \quad \cos\theta = \frac{4}{5}$

$\therefore \quad \frac{1-\cos^2\theta}{1+\cos^2\theta} = \frac{1-\left(\frac{4}{5}\right)^2}{1+\left(\frac{4}{5}\right)^2} = \frac{1-\frac{16}{25}}{1+\frac{16}{25}} = \frac{25-16}{25+16} = \frac{9}{41}$

6. **If $\tan A = \frac{3}{4}$, find the value of $\frac{1}{\sin A}+\frac{1}{\cos A}$.** *[CBSE Sample Paper 2021]*

Sol. $\tan A = \frac{3}{4} = \frac{3k}{4k}$ ½

$\sin A = \frac{3k}{5k} = \frac{3}{5}$, $\cos A = \frac{4k}{5k} = \frac{4}{5}$ ½

$\frac{1}{\sin A}+\frac{1}{\cos A} = \frac{5}{3}+\frac{5}{4}$ ½

$= \frac{20+15}{12} = \frac{35}{12}$ ½

[CBSE Marking Scheme 2021]

Short Answer Questions–II

Each of the following questions are of 3 marks.

1. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

[*NCERT Exemplar, CBSE* 2020(30/1/1)]

Sol. $\sin\theta + \cos\theta = \sqrt{3}$

$\Rightarrow$ $(\sin\theta + \cos\theta)^2 = 3$

$\Rightarrow$ $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow$ $2\sin\theta\cos\theta = 2$ $\quad (\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow$ $\sin\theta \,.\, \cos\theta = 1 = \sin^2\theta + \cos^2\theta$

$\Rightarrow$ $1 = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ $\Rightarrow$ $1 = \dfrac{\sin^2\theta}{\sin\theta\cos\theta} + \dfrac{\cos^2\theta}{\sin\theta\cos\theta}$

$\Rightarrow$ $1 = \tan\theta + \cot\theta$

Therefore $\tan\theta + \cot\theta = 1$.

2. Prove that: $\sqrt{\dfrac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\dfrac{\sec\theta + 1}{\sec\theta - 1}} = 2\,\text{cosec}\,\theta$ [*CBSE* 2019 (30/4/2)]

Sol. $\text{LHS} = \sqrt{\dfrac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\dfrac{\sec\theta + 1}{\sec\theta - 1}} = \dfrac{(\sec\theta - 1) + (\sec\theta + 1)}{\sqrt{\sec^2\theta - 1}}$ **1**

$= \dfrac{2\sec\theta}{\tan\theta}$ **1**

$= \dfrac{2}{\sin\theta} = 2\,\text{cosec}\,\theta = \text{RHS}$ **1**

[*CBSE Marking Scheme* 2019 (30/4/2)]

3. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, prove that $\tan\theta = 1$ or $\dfrac{1}{2}$. [*CBSE* 2020 (30/2/2)]

Sol. Given, $1 + \sin^2\theta = 3\sin\theta.\cos\theta$

Divide both sides by $\cos^2\theta$, we have

$$\frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{3\sin\theta.\cos\theta}{\cos^2\theta}$$

$\Rightarrow$ $\sec^2\theta + \tan^2\theta = 3\tan\theta$

$\Rightarrow$ $1 + \tan^2\theta + \tan^2\theta = 3\tan\theta$

$\Rightarrow$ $2\tan^2\theta - 3\tan\theta + 1 = 0$

$\Rightarrow$ $2\tan^2\theta - 2\tan\theta - \tan\theta + 1 = 0$

$\Rightarrow$ $2\tan\theta(\tan\theta - 1) - 1(\tan\theta - 1) = 0$

$\Rightarrow$ $(\tan\theta - 1)(2\tan\theta - 1) = 0$

$\Rightarrow$ $\tan\theta - 1 = 0$ or $2\tan\theta - 1 = 0$

$\Rightarrow$ $\tan\theta = 1$ or $2\tan\theta = 1$ $\Rightarrow$ $\tan\theta = \dfrac{1}{2}$

$\Rightarrow$ $\tan\theta = 1$ or $\dfrac{1}{2}$

4. Prove that : $(1 + \cot A - \text{cosec}\,A)(1 + \tan A + \sec A) = 2$ [*CBSE* 2019 (30/1/2)]

Sol. LHS $= (1 + \cot A - \text{cosec}\,A)\,(1 + \tan A + \sec A)$

$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$$

$$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$$

$$= \frac{1}{\sin A \cos A}[(\sin A + \cos A - 1)(\sin A + \cos A + 1)]$$

$$= \frac{1}{\sin A \cos A}\left[(\sin A + \cos A)^2 - 1\right]$$

$$= \frac{1}{\sin A \cos A}\left[\sin^2 A + \cos^2 A + 2\sin A \cos A - 1\right]$$

$$= \frac{1}{\sin A \cos A}(1 + 2\sin A \cos A - 1)$$

$$= \frac{2\sin A \cos A}{\sin A \cos A} = 2 = \text{RHS.}$$

5. Prove that : $\frac{\sin\theta - \cos\theta + 1}{\cos\theta + \sin\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$ [*CBSE* 2020 (30/4/1)]

Sol. LHS $= \frac{\sin\theta - \cos\theta + 1}{\cos\theta + \sin\theta - 1}$

Dividing N^r and D^r by $\cos\theta$

$$= \frac{\tan\theta - 1 + \sec\theta}{1 + \tan\theta - \sec\theta}$$ 1

$$= \frac{\tan\theta + \sec\theta - 1}{(\sec^2\theta - \tan^2\theta) + \tan\theta - \sec\theta}$$ 1

$$= \frac{\tan\theta + \sec\theta - 1}{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta - 1)}$$

$$= \frac{1}{\sec\theta - \tan\theta} = \text{RHS}$$ 1

[*CBSE Marking Scheme* 2020(30/4/1)]

6. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, show that $q(p^2 - 1) = 2p$. [*CBSE* 2020 (30/3/1)]

Sol. Given,

$\sin\theta + \cos\theta = p$ ...(*i*)

Squaring on both sides, we have

$(\sin\theta + \cos\theta)^2 = p^2$

$\sin^2\theta + \cos^2\theta + 2\sin\theta \cdot \cos\theta = p^2$

$\Rightarrow \quad 1 + 2\sin\theta \cdot \cos\theta = p^2$

$\Rightarrow \quad \sin\theta\cos\theta = \frac{p^2 - 1}{2}$ ...(*ii*)

Also, $\sec\theta + \operatorname{cosec}\theta = q$

$\Rightarrow \quad \frac{1}{\cos\theta} + \frac{1}{\sin\theta} = q \quad \Rightarrow \quad \frac{\sin\theta + \cos\theta}{\sin\theta \cdot \cos\theta} = q$

$\Rightarrow \quad \frac{p}{\frac{p^2 - 1}{2}} = q \quad \Rightarrow \quad \frac{2p}{p^2 - 1} = q$

$\Rightarrow \quad q(p^2 - 1) = 2p$ Proved

7. Prove that: $\dfrac{1 + \cos\theta - \sin^2\theta}{\sin\theta\,(1 + \cos\theta)} = \cot\theta$

Sol. LHS = $\dfrac{1 + \cos\theta - \sin^2\theta}{\sin\theta\,(1 + \cos\theta)}$

To obtain cot θ in RHS, we have to convert the numerator of LHS in cosine function and denominator in sin function.

Therefore converting $\sin^2\theta = 1 - \cos^2\theta$, we get

$$= \frac{1 + \cos\theta - (1 - \cos^2\theta)}{\sin\theta\,(1 + \cos\theta)} = \frac{1 + \cos\theta - 1 + \cos^2\theta}{\sin\theta(1 + \cos\theta)} = \frac{\cos\theta + \cos^2\theta}{\sin\theta\,(1 + \cos\theta)}$$

$$= \frac{\cos\theta\,(\cos\theta + 1)}{\sin\theta\,(1 + \cos\theta)} = \frac{\cos\theta}{\sin\theta} = \cot\theta = \text{RHS}$$

8. Prove that: $(\sin\theta + 1 + \cos\theta)(\sin\theta - 1 + \cos\theta).\sec\theta\,\mathrm{cosec}\,\theta = 2$ [*CBSE* 2019 (30/4/2)]

Sol. LHS $= (\sin\theta + 1 + \cos\theta)(\sin\theta - 1 + \cos\theta).\sec\theta\,\mathrm{cosec}\,\theta$

$= (\sin\theta + \cos\theta + 1)(\sin\theta + \cos\theta - 1).\sec\theta\,\mathrm{cosec}\,\theta$

$= \{(\sin\theta + \cos\theta)^2 - (1)^2\}.\sec\theta\,\mathrm{cosec}\,\theta$

$= \{\sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta - 1\}.\sec\theta\,\mathrm{cosec}\,\theta$

$= (1 + 2\sin\theta\cos\theta - 1) \times \dfrac{1}{\cos\theta.\sin\theta}$

$= 2\sin\theta.\cos\theta \times \dfrac{1}{\sin\theta.\cos\theta} = 2 = \text{RHS}$

9. If $\sec\theta = x + \dfrac{1}{4x}$, prove that $\sec\theta + \tan\theta = 2x$ or $\dfrac{1}{2x}$.

Sol. Let $\sec\theta + \tan\theta = \lambda$ ...(*i*)

We know that, $\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1 \Rightarrow \lambda(\sec\theta - \tan\theta) = 1$

$\sec\theta - \tan\theta = \dfrac{1}{\lambda}$ ...(*ii*)

Adding equations (*i*) and (*ii*), we get

$2\sec\theta = \lambda + \dfrac{1}{\lambda} \Rightarrow 2\left(x + \dfrac{1}{4x}\right) = \lambda + \dfrac{1}{\lambda}$

$\Rightarrow 2x + \dfrac{1}{2x} = \lambda + \dfrac{1}{\lambda}$

On comparing, we get $\lambda = 2x$ or $\lambda = \dfrac{1}{2x}$

$\Rightarrow \sec\theta + \tan\theta = 2x$ or $\dfrac{1}{2x}$

Alternative Method:

We have $\sec\theta = x + \dfrac{1}{4x}$

$\tan^2\theta = \sec^2\theta - 1$

$= x^2 + \dfrac{1}{16x^2} + \dfrac{1}{2} - 1$

$$= x^2 + \frac{1}{16x^2} - \frac{1}{2} = \left(x - \frac{1}{4x}\right)^2$$

$$\Rightarrow \qquad \tan\theta = \pm\left(x - \frac{1}{4x}\right)$$

$\sec\theta + \tan\theta$ is given by

$$x + \frac{1}{4x} + x - \frac{1}{4x} \quad \text{or} \quad x + \frac{1}{4x} - x + \frac{1}{4x}$$

$$= 2x \quad \text{or} \quad \frac{1}{2x}$$

10. If $\tan(A + B) = 1$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$, $0° < A + B < 90°$, $A > B$, then find the values of A and B. [*CBSE* 2019(30/3/3)]

Sol.

13. Given $\tan(A+B) = 1$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$

$\tan(A+B) = 1.$

$\Rightarrow \tan(A+B) = \tan 45°$

$\Rightarrow \quad A+B = 45°$ —— ①

Now taking,

$\tan(A-B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A-B)° = \tan 30$

$\Rightarrow \quad A-B = 30$ —— ②

Adding ① and ②;

$A+B+A-B = 45° + 30°$

$\Rightarrow \quad 2A = 75° \Rightarrow A = \frac{75}{2}° \Rightarrow A = 37.5°$

$B = 45° - A \Rightarrow B = 45° - 37.5° \Rightarrow B = 7.5°$

$\therefore$ $\boxed{A = 37.5°, \; B = 7.5°}$

[*Topper's Answer* 2019]

Long Answer Questions

Each of the following questions are of 5 marks.

1. Prove that: $\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta\,\text{cosec}\,\theta - 2\sin\theta\cos\theta$ [*CBSE* 2019 (30/3/3)]

Sol. LHS $= \dfrac{\tan^3\theta}{1+\tan^2\theta} + \dfrac{\cot^3\theta}{1+\cot^2\theta}$

$$= \frac{\dfrac{\sin^3\theta}{\cos^3\theta}}{1+\dfrac{\sin^2\theta}{\cos^2\theta}} + \frac{\dfrac{\cos^3\theta}{\sin^3\theta}}{1+\dfrac{\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\frac{\sin^3\theta}{\cos^3\theta}\times\cos^2\theta}{\cos^2\theta+\sin^2\theta} + \frac{\frac{\cos^3\theta}{\sin^3\theta}\times\sin^2\theta}{\sin^2\theta+\cos^2\theta}$$

$$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta}$$

$$= \frac{\sin^4\theta+\cos^4\theta}{\sin\theta\cos\theta} = \frac{(\sin^2\theta+\cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta} = \sec\theta\,\text{cosec}\,\theta - 2\sin\theta\cos\theta$$

= RHS Proved

2. Prove that : $\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A} = \frac{1}{1-2\cos^2 A}$ [*CBSE* 2019 (30/5/1)]

Sol. LHS $= \frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A}$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A} - 1} + \frac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A} - \frac{1}{\sin^2 A}}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A}} + \frac{\frac{1}{\sin^2 A}}{\frac{\sin^2 A - \cos^2 A}{\sin^2 A.\cos^2 A}}$$

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\cos^2 A}{\sin^2 A - \cos^2 A} = \frac{\sin^2 A + \cos^2 A}{\sin^2 A - \cos^2 A} = \frac{1}{\sin^2 A - \cos^2 A}$$

$$= \frac{1}{1 - \cos^2 A - \cos^2 A} = \frac{1}{1 - 2\cos^2 A} = \text{RHS}$$

3. If $\tan A = n \tan B$ and $\sin A = m \sin B$, prove that $\cos^2 A = \frac{m^2 - 1}{n^2 - 1}$. [*Competency Based Question*]

Sol. We have to find $\cos^2 A$ in terms of m and n. This means that the angle B is to be eliminated from the given relations.

Now, $\tan A = n \tan B$

$\Rightarrow \quad \tan B = \frac{1}{n}\tan A \quad \Rightarrow \quad \cot B = \frac{n}{\tan A}$

and $\quad \sin A = m \sin B$

$\Rightarrow \quad \sin B = \frac{1}{m}\sin A \quad \Rightarrow \quad \text{cosec}\, B = \frac{m}{\sin A}$

Substituting the values of $\cot B$ and $\text{cosec}\, B$ in $\text{cosec}^2 B - \cot^2 B = 1$, we get

$\Rightarrow \quad \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1 \quad \Rightarrow \quad \frac{m^2}{\sin^2 A} - \frac{n^2\cos^2 A}{\sin^2 A} = 1$

$\Rightarrow \quad \frac{m^2 - n^2\cos^2 A}{\sin^2 A} = 1 \quad \Rightarrow \quad m^2 - n^2\cos^2 A = \sin^2 A$

$\Rightarrow \quad m^2 - n^2\cos^2 A = 1 - \cos^2 A$

$$\Rightarrow \quad m^2 - 1 = n^2 \cos^2 A - \cos^2 A$$

$$\Rightarrow \quad m^2 - 1 = (n^2 - 1) \cos^2 A$$

$$\Rightarrow \quad \frac{m^2 - 1}{n^2 - 1} = \cos^2 A$$

4. **Prove that: $(\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = (1 + \sec\theta \operatorname{cosec}\theta)^2$**

Sol. LHS $= (\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2$

$$= \left(\sin\theta + \frac{1}{\cos\theta}\right)^2 + \left(\cos\theta + \frac{1}{\sin\theta}\right)^2 = \left(\frac{\sin\theta\cos\theta + 1}{\cos\theta}\right)^2 + \left(\frac{\cos\theta\sin\theta + 1}{\sin\theta}\right)^2$$

$$= \frac{(\sin\theta\cos\theta + 1)^2}{\cos^2\theta} + \frac{(\cos\theta\sin\theta + 1)^2}{\sin^2\theta}$$

$$= (\sin\theta\cos\theta + 1)^2\left(\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}\right)$$

$$= (\sin\theta\cos\theta + 1)^2\left(\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}\right)$$

$$= (\sin\theta\cos\theta + 1)^2.\left(\frac{1}{\cos^2\theta\sin^2\theta}\right)$$

$$= \left(\frac{\sin\theta\cos\theta + 1}{\cos\theta\sin\theta}\right)^2 = \left(1 + \frac{1}{\cos\theta\sin\theta}\right)^2$$

$$= (1 + \sec\theta\operatorname{cosec}\theta)^2 = \text{RHS.}$$

5. **Prove that : $\dfrac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{(\sec^3\theta-\operatorname{cosec}^3\theta)} = \sin^2\theta\cos^2\theta$** [*CBSE* 2019 (30/5/1)]

Sol. LHS $= \dfrac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{(\sec^3\theta-\operatorname{cosec}^3\theta)}$

$$= \frac{\left(1 + \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)(\sin\theta - \cos\theta)}{\left(\frac{1}{\cos^3\theta} - \frac{1}{\sin^3\theta}\right)}$$

$$= \frac{\frac{(\sin\theta.\cos\theta + \cos^2\theta + \sin^2\theta)(\sin\theta - \cos\theta)}{\sin\theta.\cos\theta}}{\frac{\sin^3\theta - \cos^3\theta}{\sin^3\theta.\cos^3\theta}}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{\sin\theta.\cos\theta} \times \frac{\sin^3\theta.\cos^3\theta}{\sin^3\theta - \cos^3\theta}$$

$$= \sin^2\theta.\cos^2\theta = \text{RHS}$$

6. **If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $(m^2 - n^2) = 4\sqrt{mn}$.**

[*Competency Based Question*]

Sol. We have, given $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then

LHS $= (m^2 - n^2) = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$$= \tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta - \tan^2\theta - \sin^2\theta + 2\tan\theta\sin\theta$$

$$= 4 \tan\theta \sin\theta = 4\sqrt{\tan^2\theta \sin^2\theta}$$

$$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta}(1-\cos^2\theta)} = 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$$

$$= 4\sqrt{\tan^2\theta - \sin^2\theta} = 4\sqrt{(\tan\theta - \sin\theta)(\tan\theta + \sin\theta)} = 4\sqrt{mn} = \text{RHS}$$

7. Prove that: $\dfrac{1}{(\operatorname{cosec} x + \cot x)} - \dfrac{1}{\sin x} = \dfrac{1}{\sin x} - \dfrac{1}{(\operatorname{cosec} x - \cot x)}$

Sol. In order to show that,

$$\frac{1}{(\operatorname{cosec} x + \cot x)} - \frac{1}{\sin x} = \frac{1}{\sin x} - \frac{1}{(\operatorname{cosec} x - \cot x)}$$

It is sufficient to show

$$\frac{1}{\operatorname{cosec} x + \cot x} + \frac{1}{\operatorname{cosec} x - \cot x} = \frac{1}{\sin x} + \frac{1}{\sin x}$$

$$\Rightarrow \quad \frac{1}{(\operatorname{cosec} x + \cot x)} + \frac{1}{(\operatorname{cosec} x - \cot x)} = \frac{2}{\sin x} \qquad \text{...}(i)$$

Now, LHS of above is

$$\frac{1}{(\operatorname{cosec} x + \cot x)} + \frac{1}{(\operatorname{cosec} x - \cot x)}$$

$$= \frac{(\operatorname{cosec} x - \cot x) + (\operatorname{cosec} x + \cot x)}{(\operatorname{cosec} x - \cot x)(\operatorname{cosec} x + \cot x)}$$

$$= \frac{2 \operatorname{cosec} x}{\operatorname{cosec}^2 x - \cot^2 x} \qquad (\because (a+b)(a-b) = a^2 - b^2)$$

$$= \frac{2 \operatorname{cosec} x}{1} = \frac{2}{\sin x} = \text{RHS of } (i)$$

Hence, $\dfrac{1}{(\operatorname{cosec} x + \cot x)} + \dfrac{1}{(\operatorname{cosec} x - \cot x)} = \dfrac{1}{\sin x} + \dfrac{1}{\sin x}$

or $\dfrac{1}{(\operatorname{cosec} x + \cot x)} - \dfrac{1}{\sin x} = \dfrac{1}{\sin x} - \dfrac{1}{(\operatorname{cosec} x - \cot x)}$.

8. If $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$ and $x \sin\theta = y \cos\theta$, prove $x^2 + y^2 = 1$.

[*Competency Based Question*]

Sol. We have, $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$

$\Rightarrow \quad (x \sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$

$\Rightarrow \quad x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta \qquad (\because x\sin\theta = y\cos\theta)$

$\Rightarrow \quad x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$

$\Rightarrow \quad x\sin\theta = \sin\theta\cos\theta$

$\Rightarrow \quad x = \cos\theta$

Now, we have $x\sin\theta = y\cos\theta$

$\Rightarrow \quad \cos\theta\sin\theta = y\cos\theta \qquad (\because x = \cos\theta)$

$\Rightarrow \quad y = \sin\theta$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$.

PROFICIENCY EXERCISE

■ Objective Type Questions: [1 mark each]

1. Choose and write the correct option in each of the following questions.

(*i*) Considering the diagram below.

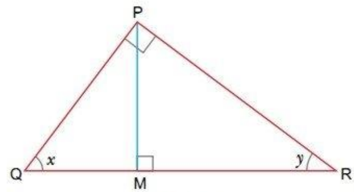

Fig. 9.10

Which of the following statements is true?

(*a*) Side PR is adjacent to $\angle y$ in triangle PMR and side QR is adjacent to $\angle y$ in triangle PQR.

(*b*) Side MR is adjacent to $\angle y$ in triangle PMR and side PR is adjacent to $\angle y$ in triangle PQR.

(*c*) Side PR is adjacent to $\angle y$ in triangle PMR and side MR is adjacent to $\angle y$ in triangle PQR.

(*d*) Side PR is adjacent to $\angle y$ in triangle PMR and triangle PQR.

(*ii*) Observe the figure shown.

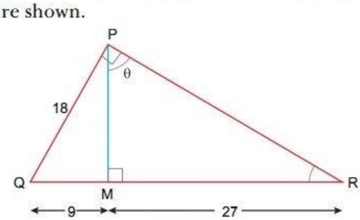

Fig. 9.11

Which of these is the value of cos θ? [*Competency Based Question*]

(*a*) $\frac{1}{2}$ (*b*) $\frac{2}{1}$ (*c*) $\frac{2\sqrt{3}}{3}$ (*d*) $\frac{3\sqrt{3}}{2}$

(*iii*) The value of $4(\sin^4 30° + \cos^4 60°) - 3(\cos^2 45° - \tan^2 45°)$ is

(*a*) $\frac{1}{2}$ (*b*) 1 (*c*) 2 (*d*) 3

(*iv*) α is an acute angle (sin α + cos α) is [*CBSE Question Bank*]

(*a*) greater than 1. (*b*) less than 1.

(*c*) equal to 1. (*d*) We cannot say any of these as it depends on the value of α.

(*v*) Which of the following option makes the statement below true?

$$\frac{\frac{1}{\sec x} + \sec x}{\cos^2 x - 1 - \tan^2 x}$$

(*a*) $-\text{cosec}\, x \tan x$ (*b*) $-\sec x \tan x$ (*c*) $-\text{cosec}\, x \cot x$ (*d*) $-\sec x \cot x$

■ Very Short Answer Questions: [1 mark each]

2. If $\sin A = \frac{3}{4}$, calculate sec A. [*CBSE* 2019 (30/2/1)]

3. If $\sin\theta = \frac{12}{13}$, then find $\tan\theta$.

4. If $\text{cosec}^2\theta\,(1+\cos\theta)(1-\cos\theta) = k$, then find the value of k. **[*CBSE* 2019(C)(30/1/1)]**

5. If $\tan\alpha = \sqrt{3}$ and $\tan\beta = \frac{1}{\sqrt{3}}$, then find the value of $\cot(\alpha+\beta)$.

6. Evaluate: $\sin^2 60° + 2\tan 45° - \cos^2 30°$ **[*CBSE* 2019 (30/2/1)]**

■ Short Answer Questions-I: [2 marks each]

7. What is the maximum value of $\frac{2}{\text{cosec}\,\theta}$? Justify your answer.

8. If $\text{cosec}\,\theta = 3x$ and $\cot\theta = \frac{3}{x}$, then find the value of $\left(x^2 - \frac{1}{x^2}\right)$.

9. What is the value of $\sin^2\theta + \frac{1}{1+\tan^2\theta}$?

■ Short Answer Questions-II: [3 marks each]

10. In ΔABC, right-angled at C, find $\cos A$, $\tan A$ and $\text{cosec}\,B$ if $\sin A = \frac{24}{25}$.

11. In Fig. 9.12, find $\sin A$, $\tan A$ and $\cot A$.

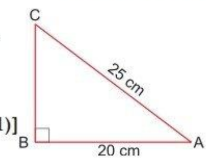

Fig. 9.12

12. If $4\tan\theta = 3$, evaluate $\left(\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right)$. **[*CBSE* 2018 (30/1)]**

13. Prove that:

$$\frac{\tan A}{1+\sec A} - \frac{\tan A}{1-\sec A} = 2\,\text{cosec}\,A$$

[*CBSE* 2019(C)(30/1/1)]

14. If $\cot\theta = \frac{1}{\sqrt{3}}$, show that $= \frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$.

15. If $\sin\theta = \frac{a^2-b^2}{a^2+b^2}$, find $1+\tan\theta\,\cos\theta$.

16. If $\sec\theta = \frac{5}{4}$, find the value of $\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$.

Prove the following identities. (Q 17 and 18)

17. $(\text{cosec}\,\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

18. $\cot\theta - \tan\theta = \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$

19. Evaluate: $\frac{\tan^2 60° + 4\cos^2 45° + 3\sec^2 30°}{\text{cosec}\,30° + \sec 60° - \cot^2 30°}$

20. Evaluate: $\frac{\tan 45°}{\text{cosec}\,30°} + \frac{\sec 60°}{\cot 45°} - \frac{3}{2}$

21. Prove that: **[*CBSE* 2019(30/5/1)]**

$$\frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{(\sec^3\theta - \text{cosec}^3\theta)} = \sin^2\theta\cos^2\theta$$

22. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = 0$, $0° < A+B \le 90°$, find $\sin(A+B)$ and $\cos(A-B)$.

Long Answer Questions: [5 marks each]

23. Prove that: $(\sec A - \text{cosec } A)(1 + \tan A + \cot A) = \tan A \sec A - \cot A \text{ cosec } A$

24. If $a \sin\theta + b\cos\theta = c$, then prove that $a\cos\theta - b\sin\theta = \sqrt{a^2 + b^2 - c^2}$.

25. Prove that: $\sin A(1 + \tan A) + \cos A(1 + \cot A) = \sec A + \text{cosec } A$

26. Prove that: $(1 + \cot\theta + \tan\theta)(\sin\theta - \cos\theta) = \frac{\sec\theta}{\text{cosec}^2\theta} - \frac{\text{cosec}\,\theta}{\sec^2\theta}$

27. If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$, prove that $x^2 - y^2 = a^2 - b^2$.

28. Prove that: [*CBSE* 2019 (30/4/3)]

$$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$$

Answers

1. (*i*) (*b*) (*ii*) (*a*) (*iii*) (*c*) (*iv*) (*a*) (*v*) (*c*)

2. $\frac{4}{\sqrt{7}}$ 3. $\frac{12}{5}$ 4. $k = 1$ 5. 0 6. 2

7. 2 8. $\frac{1}{9}$ 9. 1 10. $\cos A = \frac{7}{25}$, $\tan A = \frac{24}{7}$, $\text{cosec } B = \frac{25}{7}$

11. $\sin A = \frac{3}{5}, \tan A = \frac{3}{4}, \cot A = \frac{4}{3}$ 12. $\frac{13}{11}$ 15. $\frac{2a^2}{a^2 + b^2}$ 16. $\frac{12}{7}$

19. 9 20. 1 22. $\sin(A + B) = \frac{\sqrt{3}}{2}$, $\cos(A - B) = 1$

Self-Assessment

Time allowed: 1 hour — Max. marks: 40

SECTION A

1. **Choose and write the correct option in the following questions.** (3 × 1 = 3)

(*i*) If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta, (\theta \neq 90°)$, then the value of $\tan\theta$ is

(*a*) $\sqrt{2} - 1$ (*b*) $\sqrt{2} + 1$ (*c*) $\sqrt{2}$ (*d*) $-\sqrt{2}$

(*ii*) Given that $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is [*NCERT Exemplar*]

(*a*) 0° (*b*) 30° (*c*) 60° (*d*) 90°

(*iii*) $\frac{1}{\tan\theta + \cot\theta} =$

(*a*) $\cos\theta\sin\theta$ (*b*) $\sec\theta\sin\theta$ (*c*) $\tan\theta\cot\theta$ (*d*) $\sec\theta \text{ cosec }\theta$

2. **Solve the following questions.** (2 × 1 = 2)

(*i*) If $\tan\alpha = \frac{5}{12}$, find the value of $\sec\alpha$. [*CBSE* 2019 (30/3/1)]

(*ii*) In a right angled triangle if $\cos\theta = \frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$, what is the value of $\tan\theta$?

SECTION B

- **Solve the following questions.** $(4 \times 2 = 8)$

3. In the ΔABC shown below $\angle x : \angle y = 1:2$. What is the value of tan x? **[*Competency Based Question*]**

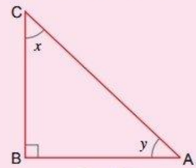

Fig. 9.13

4. Find an acute angle θ, when $\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \dfrac{1-\sqrt{3}}{1+\sqrt{3}}$.

5. Evaluate: $\dfrac{\cos 60° - \cot 45° + \text{cosec}\, 30°}{\sec 60° + \tan 45° - \sin 30°}$

6. If $A = 30°$ and $B = 30°$, verify that $\sin(A+B) = \sin A \cos B + \cos A \sin B$.

- **Solve the following questions.** $(4 \times 3 = 12)$

7. Prove that: $\dfrac{1}{\cos A + \sin A - 1} + \dfrac{1}{\cos A + \sin A + 1} = \text{cosec}\, A + \sec A$

8. Prove that: $(\text{cosec}\,\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

9. Prove that : $\dfrac{\tan\theta}{1-\tan\theta} - \dfrac{\cot\theta}{1-\cot\theta} = \dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$ **[*CBSE* 2019 (30/5/1)]**

10. Prove that : $\dfrac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$

- **Solve the following questions.** $(3 \times 5 = 15)$

11. The altitude AD of a ΔABC, in which $\angle A$ is an obtuse angle has length 10 cm. If $BD = 10$ cm and $CD = 10\sqrt{3}$ cm, determine $\angle A$.

12. Prove that: $2\sec^2\theta - \sec^4\theta - 2\,\text{cosec}^2\theta + \text{cosec}^4\theta = \cot^4\theta - \tan^4\theta$

13. Prove that: [*CBSE* 2019 (30/2/1)]

$$\frac{\sin\theta}{\cot\theta + \text{cosec}\,\theta} = 2 + \frac{\sin\theta}{\cot\theta - \text{cosec}\,\theta}$$

Answers

1. (*i*) (*a*)   (*ii*) (*d*)   (*iii*) (*a*)

2. (*i*) $\dfrac{13}{12}$ (*ii*) $\sqrt{3}$ **3.** $\dfrac{1}{\sqrt{3}}$ **4.** 60°   **5.** –1   **11.** 105°